

Numerical Analysis Syllabus

Sources:

[1] Burden & Faires, Numerical Analysis (10th edition)

<https://www.cengage.com/c/numerical-analysis-10e-faires/9781305253667/>

[2] Trefethen & Bau, Numerical Linear Algebra (25th anniversary edition)

<https://epubs.siam.org/doi/10.1137/1.9781611977165>

[3] Moin, Fundamental of Engineering Numerical Analysis (2nd edition)

<https://www.cambridge.org/core/books/fundamentals-of-engineering-numerical-analysis/D6B6B75172AD7A5A555DC506FDDA9B99>

[4] Ascher & Grief, A First Course in Numerical Methods

<https://epubs.siam.org/doi/book/10.1137/9780898719987>

1. Basic principles

1.1 Floating point arithmetic & rounding error. [1; Ch 1] or [2; Part III] or [4; Ch 2]

1.2 Conditioning & Stability. [1; Ch 1] or [2; Part III] or [4, Ch 1]

2. Root finding and optimization

2.1 One-variable: Bisection, fixed point iteration, and Newton's Method (including convergence analysis). [1; Ch. 2] or [4; Ch. 3]

2.2 Multiple variables: Gradient descent and Newton's method. [1; Ch 10]

3. Interpolation

3.1 Lagrange & Newton forms. [1; Ch. 3] or [3; Ch 1] or [4; Ch. 10]

3.2 Error in interpolation. [1; Ch. 3] or [3; Ch 1] or [4; Ch. 10]

3.3 Piecewise polynomial interpolation and splines. [1; Ch. 3] or [3; Ch 1] or [4; Ch 11]

4. Approximation

4.1 Least-squares approximation [1; Ch. 8] or [4; Ch 12]

4.2 Orthogonal polynomials [1; Ch. 8] or [4; Ch 12]

4.3 Trigonometric polynomial approximation [1; Ch. 8] or [4; Ch 13]

5. Numerical differentiation and integration

5.1 Numerical differentiation and errors in finite differences [1; Ch. 4] or [3; Ch 2] or [4; Ch 14]

5.2 Quadrature via Newton-Cotes [1; Ch. 4] or [3; Ch 3] or [4; Ch 15]

5.3. Gaussian quadrature [1; Ch. 4] or [3; Ch 3] or [4; Ch 15]

6. Numerical linear algebra

6.1 Direct solution of systems of linear equations: LU and Cholesky factorization [1; Ch. 6] or [2; Part IV] or [4; Ch 5]

6.2 Iterative techniques for solving linear systems: Gauss-Seidel, Jacobi for linear systems [1; Ch. 7] or [2; Part 5] or [4; Ch 7]

6.3 Least squares: normal equations, QR factorization [1; Ch. 9] or [2; Part II and Part V] or [4; Ch 6]

6.4 Eigenvalues and eigenvectors via power method [1; Ch. 9] or [2; Part V] or [4; Ch 8]

7. Initial-value problems for ODEs

7.1 Basic theory [1; Ch 5] or [3; Ch 4] or [4; Ch 16]

7.2 Euler's method [1; Ch 5] or [3; Ch 4] or [4; Ch 16]

7.3 Runge-Kutta [1; Ch 5] or [3; Ch 4] or [4; Ch 16]

7.4 Multi-step methods [1; Ch 5] or [3; Ch 4] or [4; Ch 16]

7.5 Convergence [1; Ch 5] or [3; Ch 4]

8. Boundary value problems for ODEs

8.1 Finite difference methods [1; Ch 11] or [3; Ch 4]

8.2 Shooting method [1; Ch 11] or [3; Ch 4]

Students should also have some experience implementing numerical algorithms on a computer. The preliminary exam will not involve implementing algorithms on a computer, but may involve writing pseudocode.

Recommended classes

Math department: MAT 128ABC and MAT 226ABC; Outside the math department: ECH 60 and ECI 115